Strength of Material

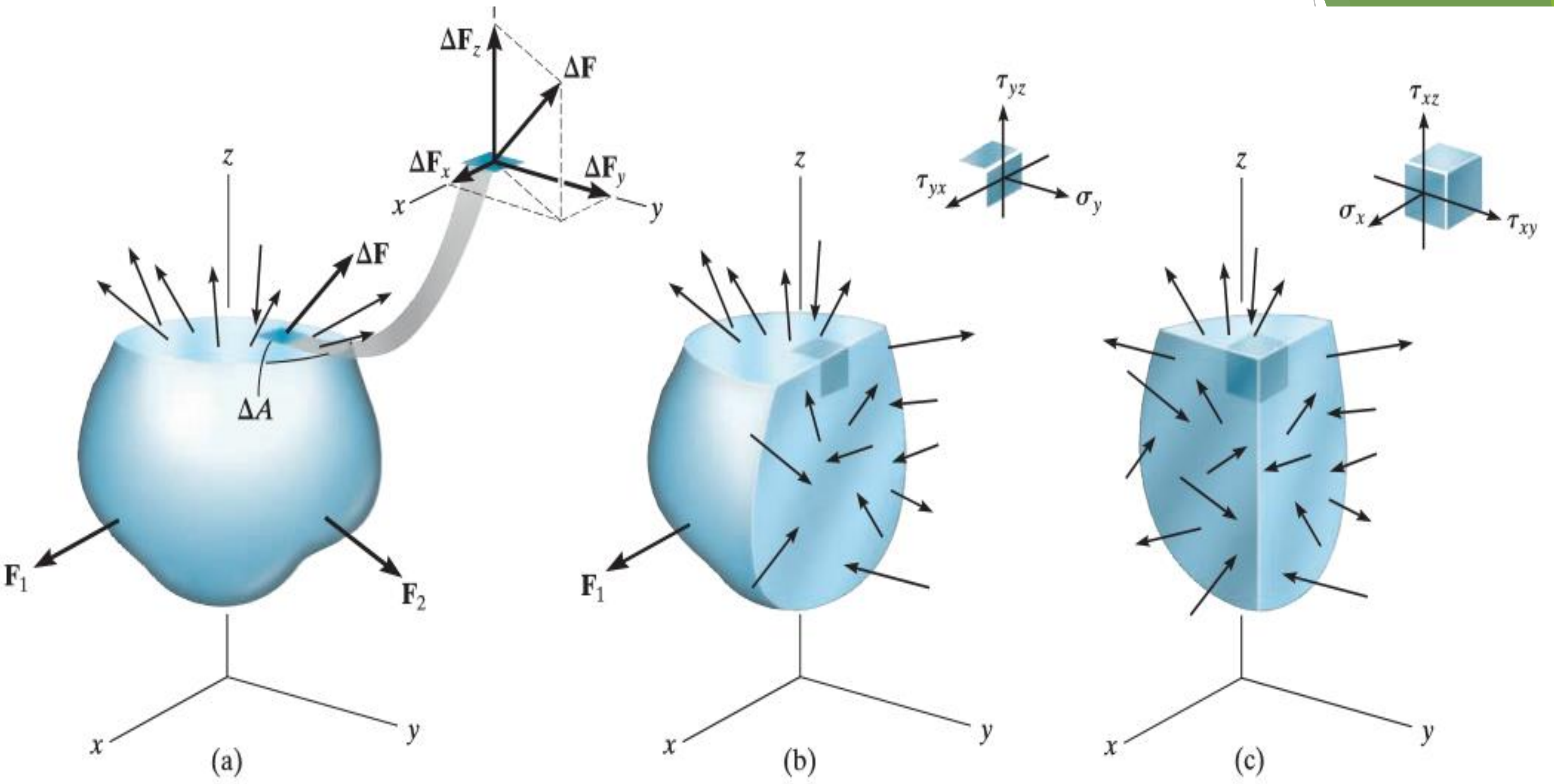
Stress

Stress

Normal Stress. The *intensity* of the force acting normal to ΔA is defined as the ***normal stress***, σ (sigma). Since $\Delta \mathbf{F}_z$ is normal to the area then

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

If the normal force or stress "pulls" on ΔA as shown in Fig. 1 , it is referred to as *tensile stress*, whereas if it "pushes" on ΔA it is called *compressive stress*.

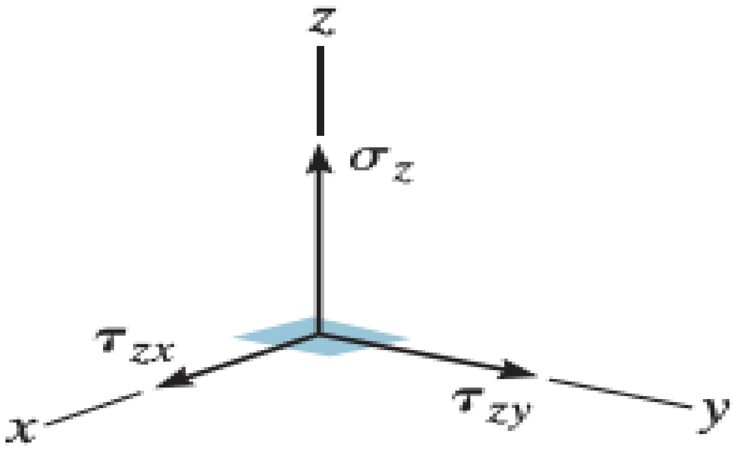


ΔF_z
ΔF
ΔF_x
ΔF_y
x
y
z
ΔF
ΔA
F_1
F_2
(a)
τ_yz
τ_yx
σ_y
F_1
(b)
τ_xz
σ_x
τ_xy
(c)

Shear Stress. The intensity of force acting tangent to ΔA is called the *shear stress*, τ (tau). Here we have shear stress components,

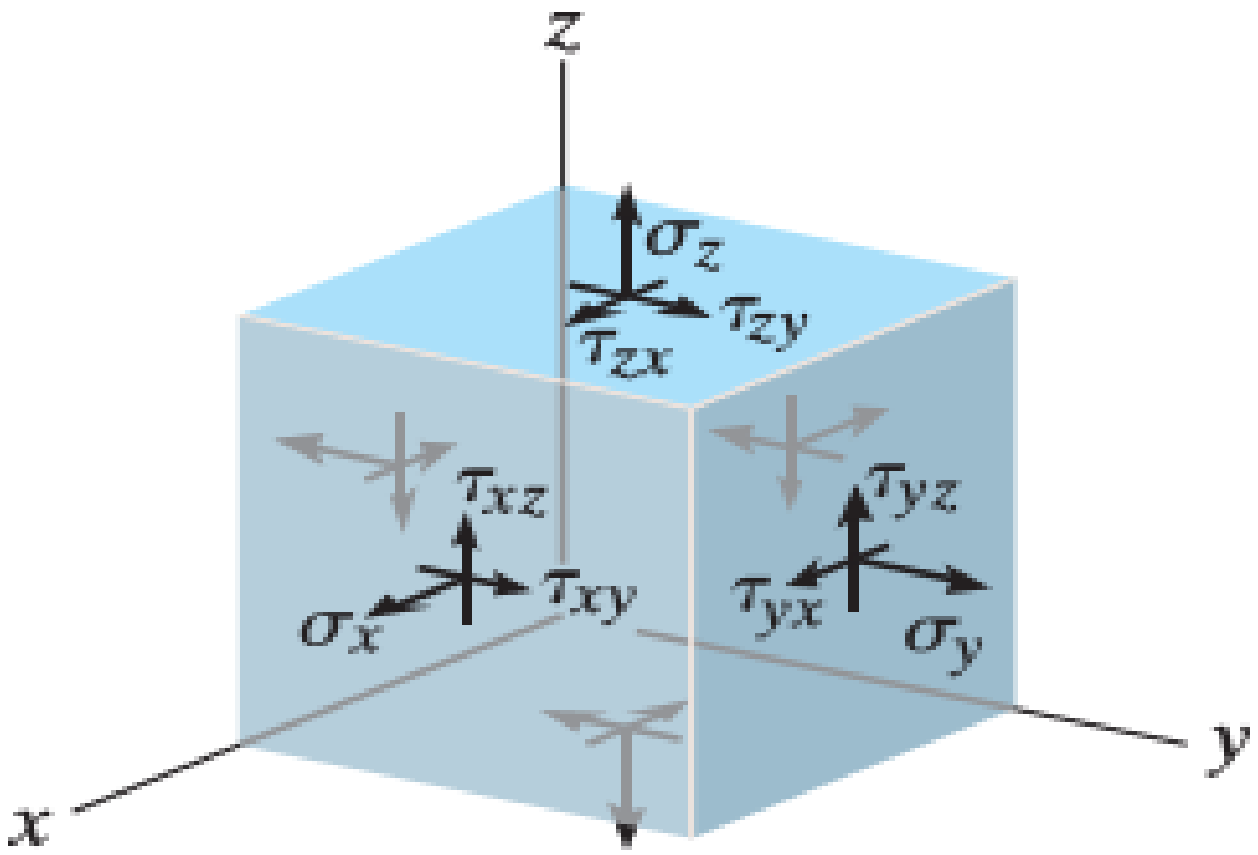
$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



Note that in this subscript notation z specifies the orientation of the area ΔA, Fig. 2 , and x and y indicate the axes along which each shear stress acts.

General State of Stress



$$\int dF = \int_A \sigma\, dA \quad \Rightarrow \quad P = \sigma A \quad \Rightarrow \quad \boxed{\sigma = \frac{P}{A}}$$

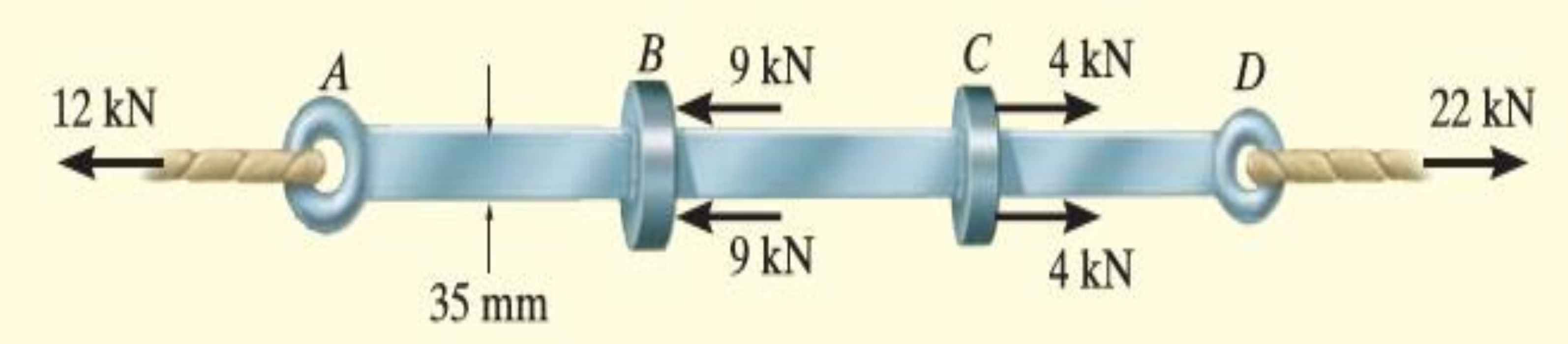
σ = average normal stress at any point on the cross-sectional area

P = *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. P is determined using the method of sections and the equations of equilibrium

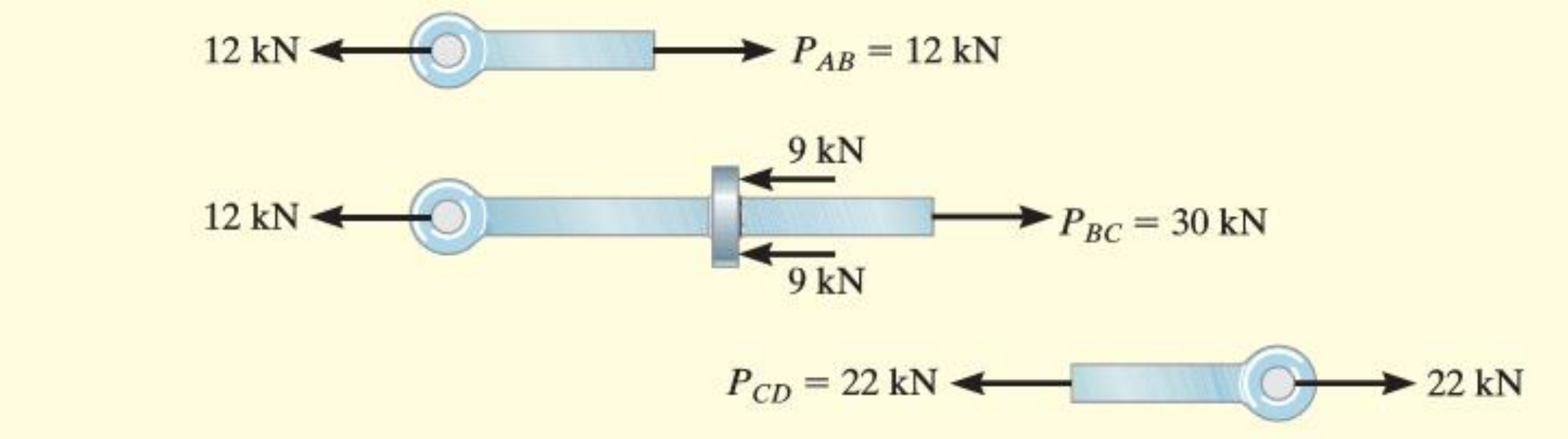
A = cross-sectional area of the bar where σ is determined

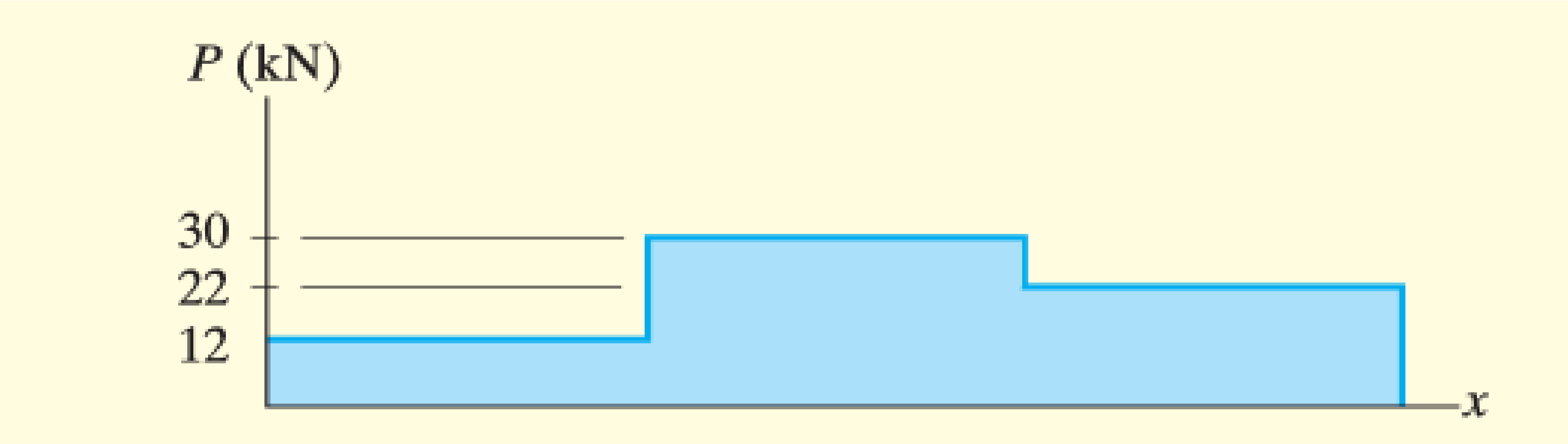
Example 1

The bar in Fig. 1–16*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



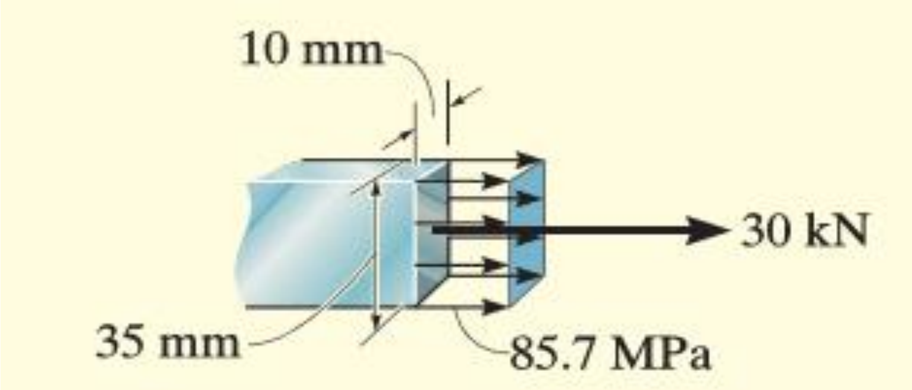
SOLUTION





Average Normal Stress.

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\,\text{N}}{(0.035\ \text{m})(0.010\ \text{m})} = 85.7\ \text{MPa} \qquad \textit{Ans.}$$



Example 2

The 80-kg lamp is supported by two rods *AB* and *BC* as shown in Fig. 1–17*a*. If *AB* has a diameter of 10 mm and *BC* has a diameter of 8 mm, determine the average normal stress in each rod.

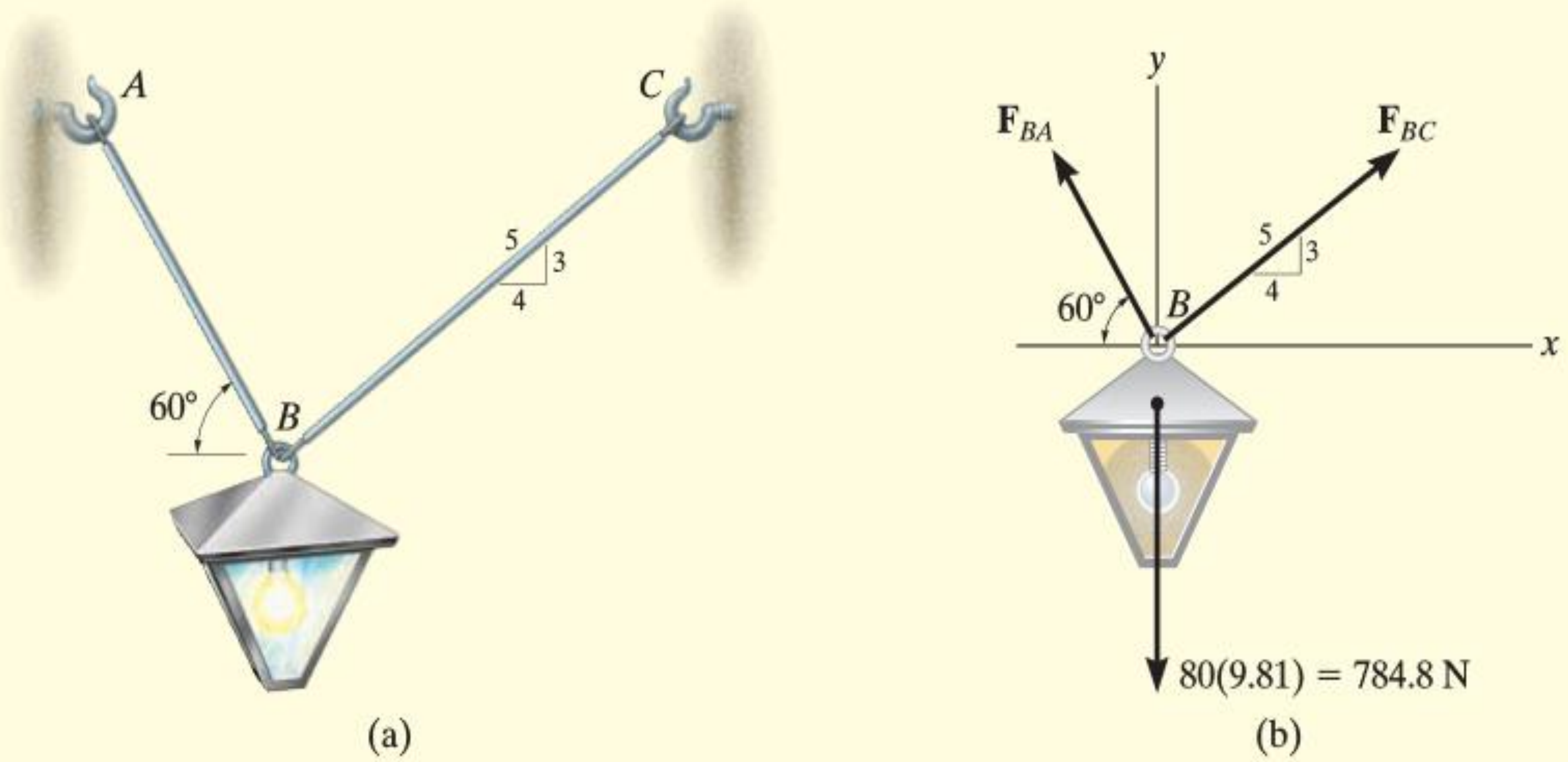


Fig. 1–17

SOLUTION

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

Average Normal Stress.

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$

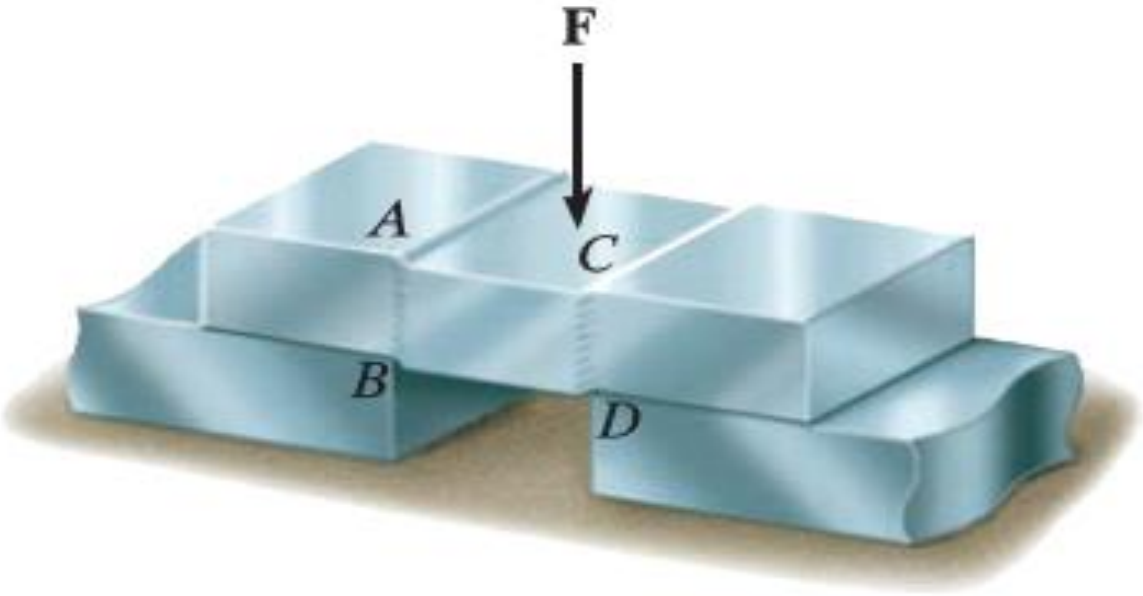
Average Shear Stress

$$\tau_{avg} = \frac{V}{A}$$

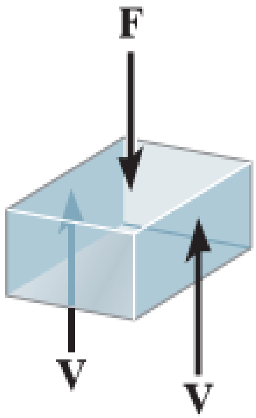
τ_{avg} = average shear stress at the section, which is assumed to be the *same* at each point located on the section

V = internal resultant shear force on the section determined from the equations of equilibrium

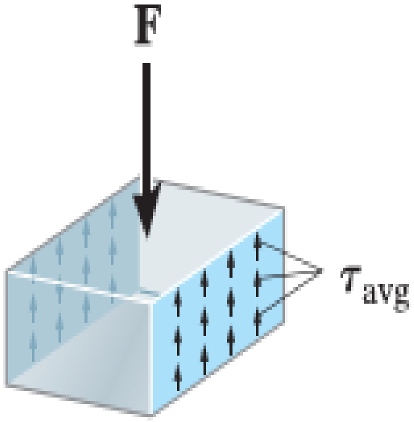
A = area at the section



(a)



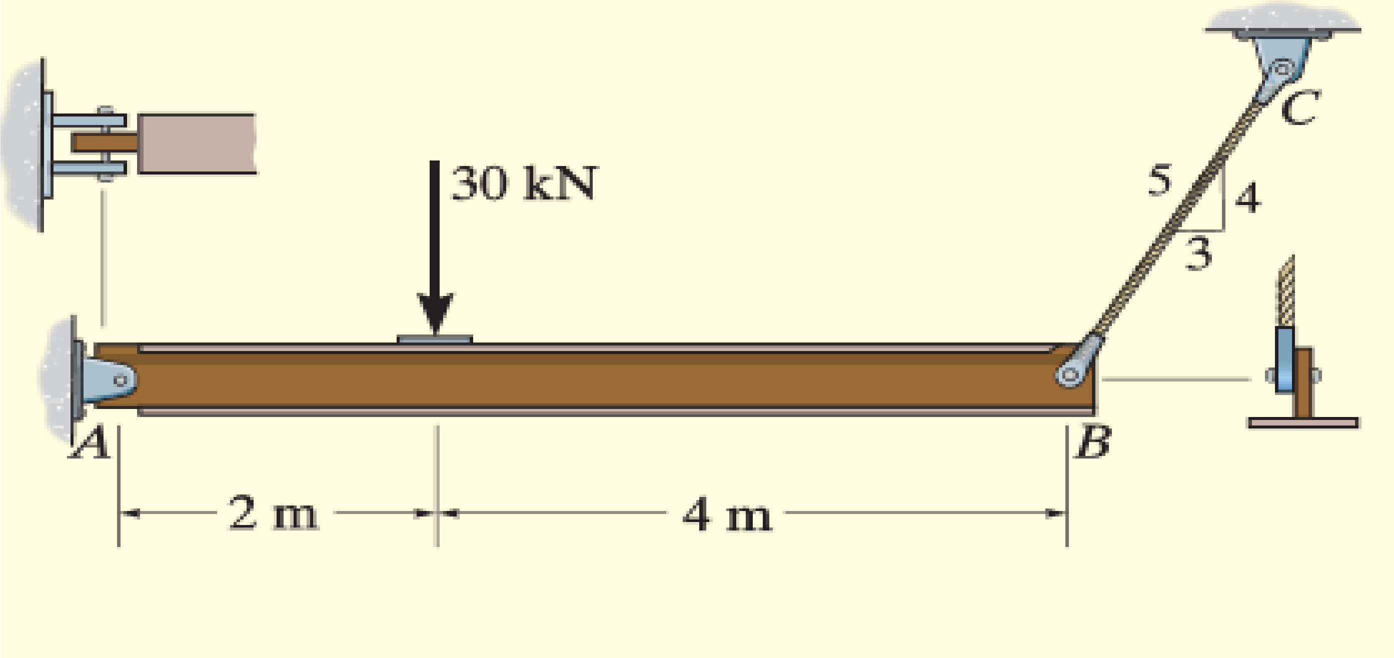
(b)



(c)

EXAMPLE

Determine the average shear stress in the 20-mm-diameter pin at *A* and the 30-mm-diameter pin at *B* that support the beam in Fig. 1–22*a*.



(a)

SOLUTION

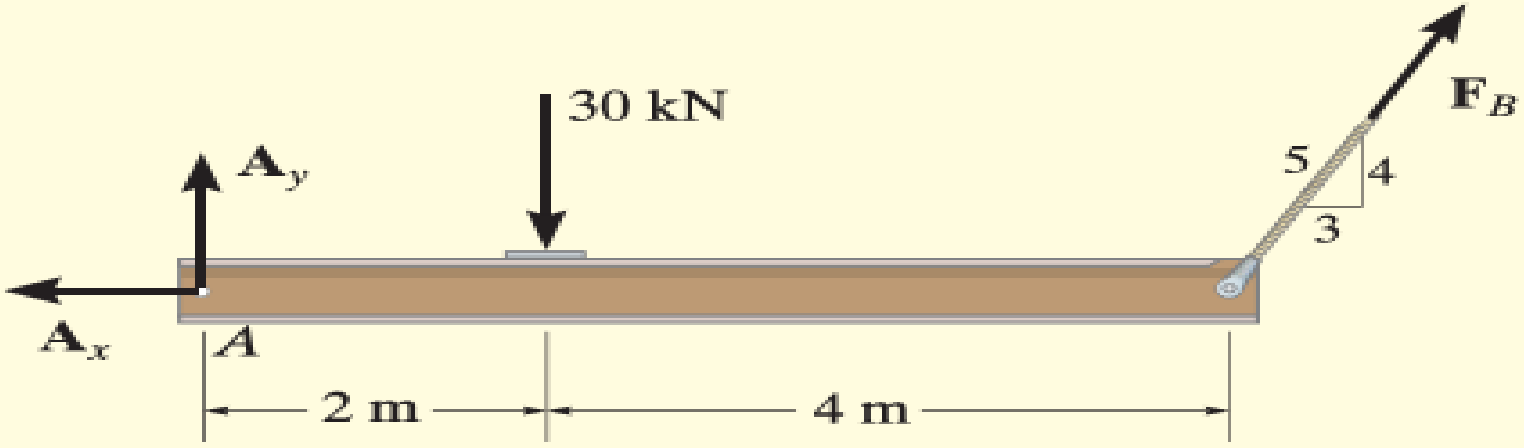
Internal Loadings. The forces on the pins can be obtained by considering the equilibrium of the beam, Fig. 1–22*b*.

$$\circlearrowleft + \Sigma M_A = 0; \quad F_B\left(\frac{4}{5}\right)(6\text{ m}) - 30\text{ kN}(2\text{ m}) = 0 \quad F_B = 12.5\text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad (12.5\text{ kN})\left(\frac{3}{5}\right) - A_x = 0 \quad A_x = 7.50\text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + (12.5\text{ kN})\left(\frac{4}{5}\right) - 30\text{ kN} = 0$$

$$A_y = 20\text{ kN}$$

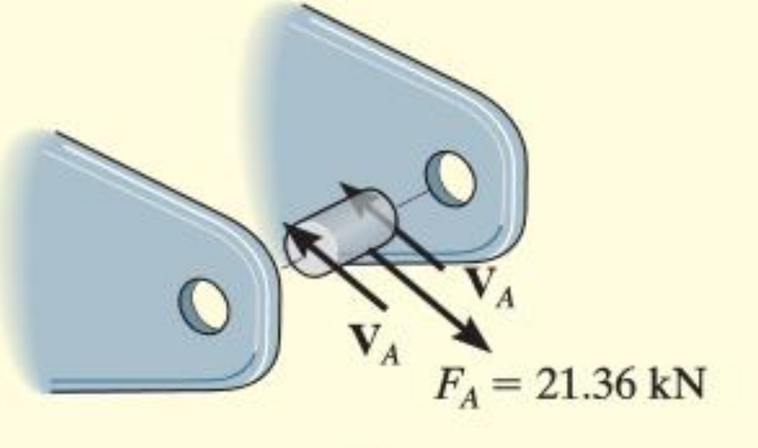


(b)

Thus, the resultant force acting on pin A is

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{(7.50\text{ kN})^2 + (20\text{ kN})^2} = 21.36\text{ kN}$$

$$V_A = \frac{F_A}{2} = \frac{21.36\text{ kN}}{2} = 10.68\text{ kN}$$



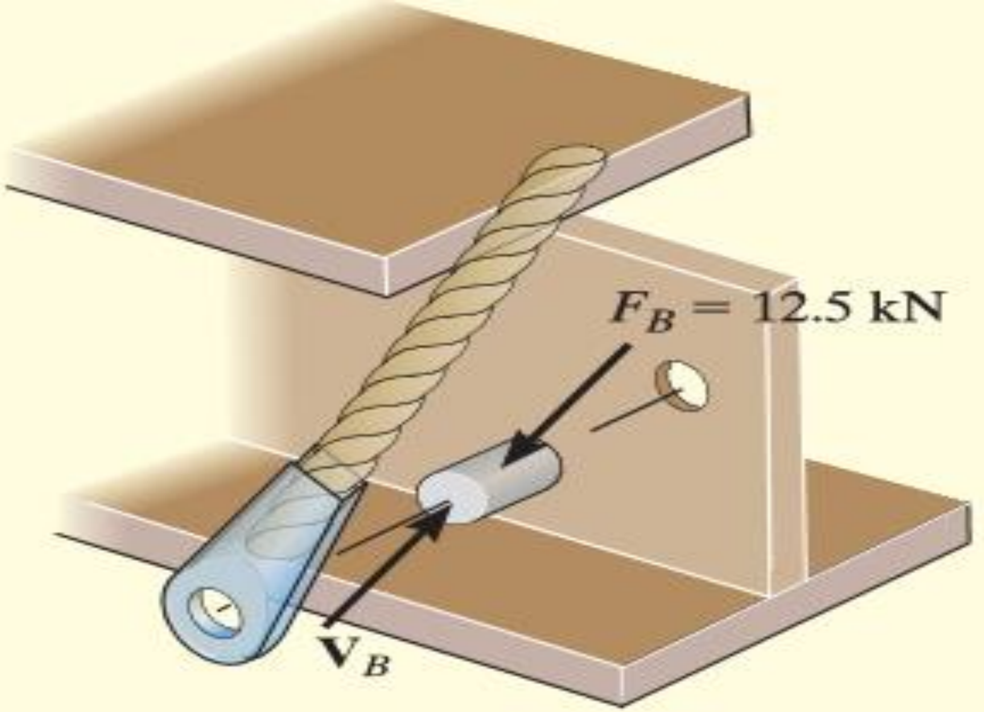
(c)

$$V_B = F_B = 12.5 \text{ kN}$$

Average Shear Stress.

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3)\text{ N}}{\frac{\pi}{4}(0.02\text{ m})^2} = 34.0\text{ MPa}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3)\text{ N}}{\frac{\pi}{4}(0.03\text{ m})^2} = 17.7\text{ MPa}$$



(d)